

10.3 Polar Coordinates

Polar

Given (r, θ)

1. Stand at origin facing the positive x -axis.

2. Rotate by θ .

pos. = ccw,

neg. = clockwise

3. Walk r -units in direction you are facing.

pos. = forward

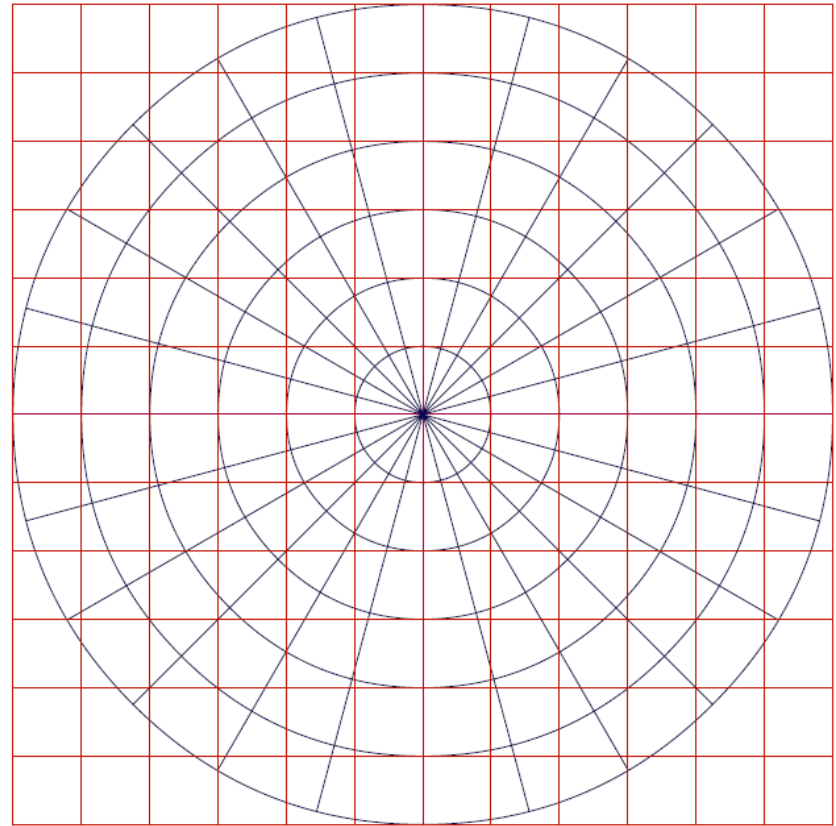
neg. = backward

Entry Task: Plot these polar points

$$(1) (r, \theta) = (2, -\pi/4)$$

$$(2) (r, \theta) = (1, 2\pi/3)$$

$$(3) (r, \theta) = (-1, \pi/4)$$



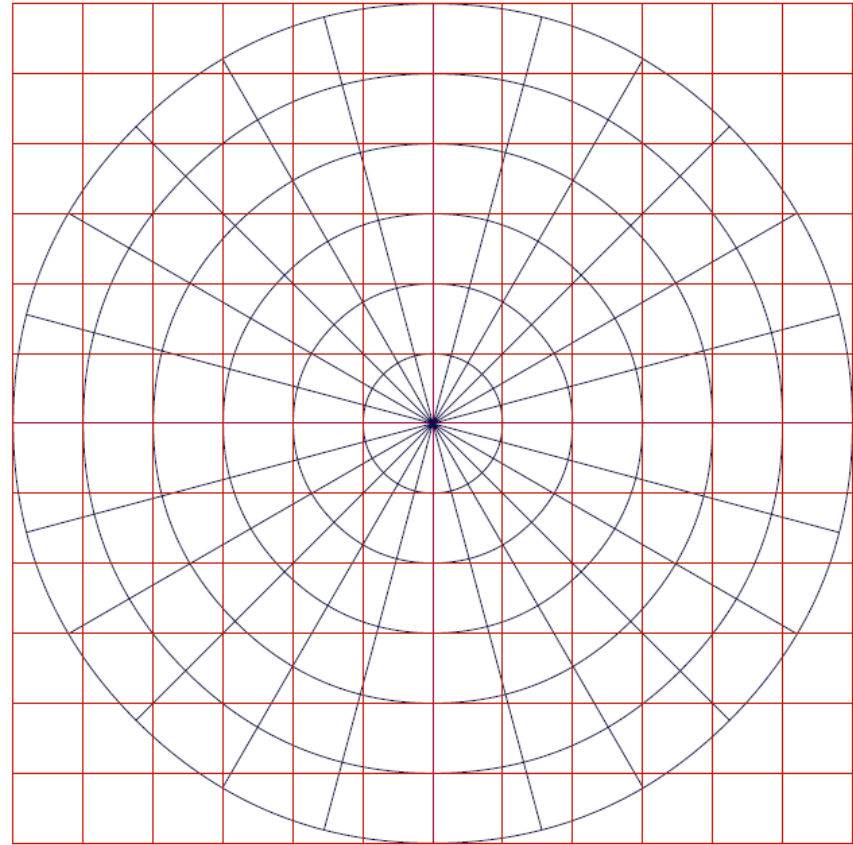
From trig we already know:

$$x = r \cos(\theta), \quad y = r \sin(\theta)$$

$$\tan(\theta) = \frac{y}{x}, \quad x^2 + y^2 = r^2$$

Exercise:

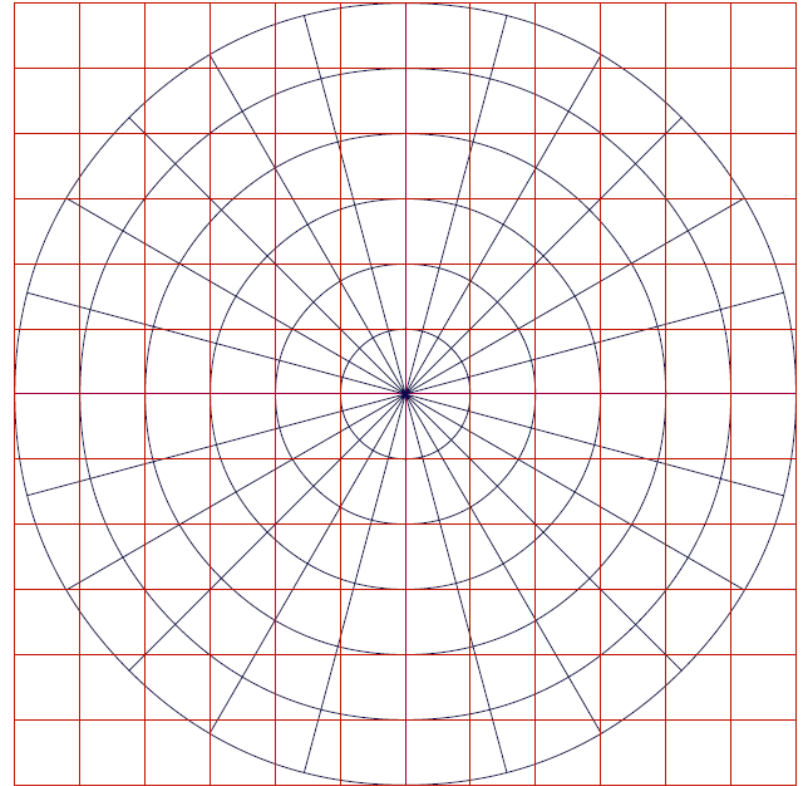
1. Describe all pts where $r = 3$.
2. Describe all pts where $\theta = \pi/4$.



Polar Regions

1. Describe all pts where
$$-\frac{\pi}{4} \leq \theta \leq \pi \text{ and } 1 \leq r \leq 3$$

2. Describe all pts where
$$0 \leq \theta \leq 2\pi \text{ and } 0 \leq r \leq 2$$



Plotting Polar Curves

Option 1: Try to convert to x and y .
Then hope you recognize the curve.

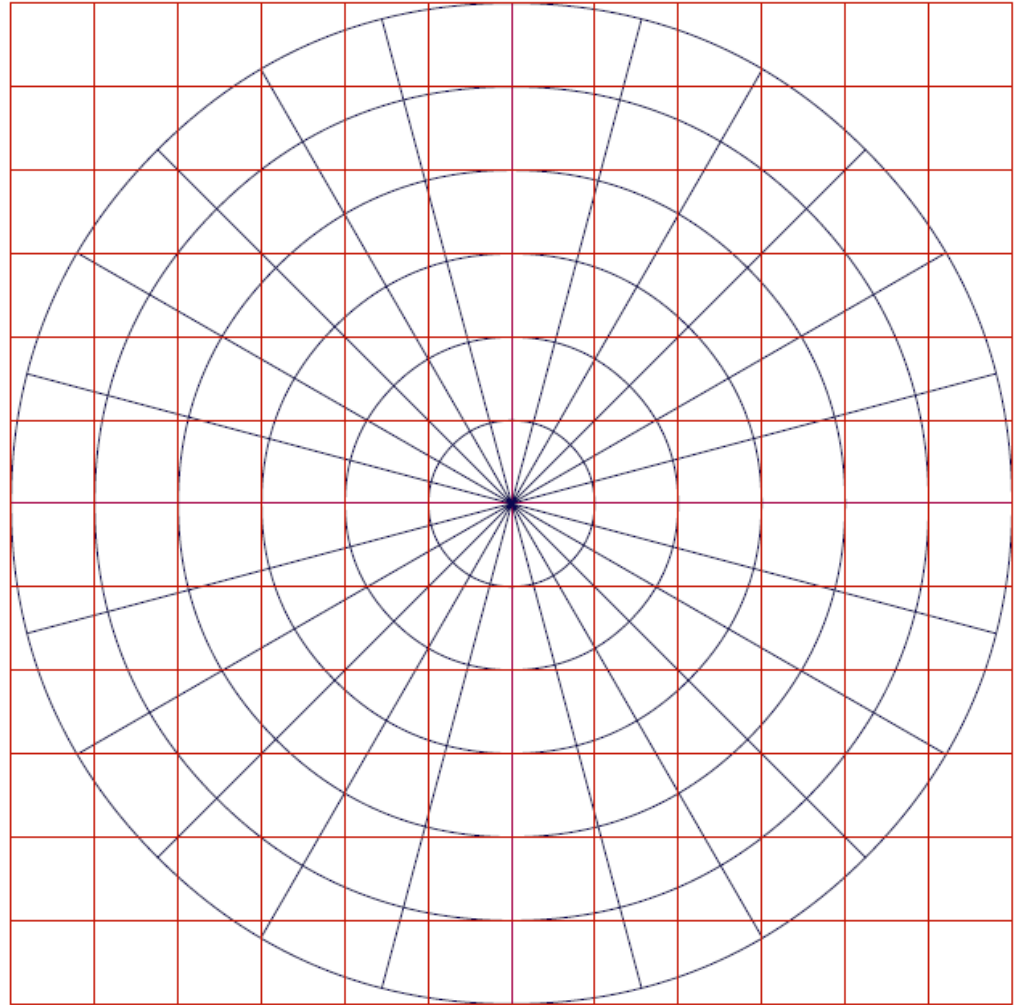
Option 2: **Plot points!**

Start with $0, \pi/2, \pi, 3\pi/2$ (intercepts).
For more detail do multiples of $\pi/6$
and $\pi/4$.

Example: Graph $r = \sin(\theta)$

θ	0	$\pi/2$	π	$3\pi/2$	2π
r					

θ	$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/3$	$3\pi/4$	$5\pi/6$
r						

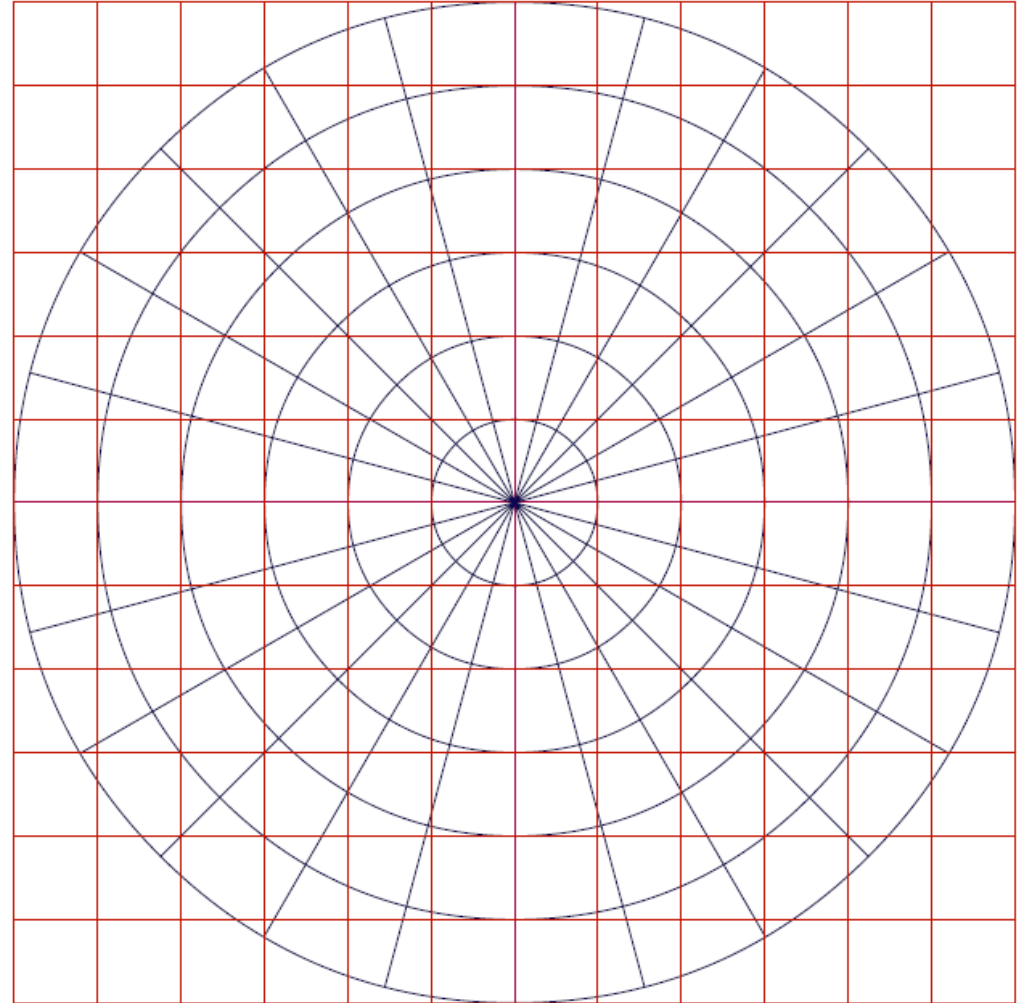


Example: Graph $r = \cos(2\theta)$

θ	0	$\pi/2$	π	$3\pi/2$	2π
r					

θ	$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/3$	$3\pi/4$	$5\pi/6$
r						

Question: Give “bounds” that describe “one loop”.



An old exam question:

The four polar equations below each match up with one of the six pictures. Identify which match.

1. $r = \sqrt{\theta}$
2. $r = 1 - 2\cos(\theta)$
3. $r = 1 + \sin(2\theta)$
4. $r = 9\cos(\theta)$

